

## Rudi Mathematici

## $x^{4}-8204 x^{3}+25237646 x^{2}-34502914684 x+17687247380985=0$



DILBERT By Scort Adrus


THE AUTHOR LIST: GIVING CREDIT WHERE CREDIT IS DUE


YOUR THESIS TITLE CONDENSNG OVER HALF ADECADE of

| the colon <br> Can't decide what to title your thesis? Use a colon |  |  <br>  |  |
| :---: | :---: | :---: | :---: |
| "Witty catchphrase" | Length-enhanced superlative verbiage with prolixity | $\begin{gathered} \text { in/of/ } / \\ \text { for } \end{gathered}$ | Obscure topic few people care about |
|  |  |  | obscure topic few people care about Sad, but true. |



Rudi Mathematici January


## USAMO 1999 - Pr. 1

Some checkers placed on an $n \times n$ checkerboard satisfy the following conditions:
(a) every square that does not contain a checker shares a side with one that does;
(b) given any pair of squares that contain checkers, there is a sequence of squares containing checkers, starting and ending with the given squares, such that every two consecutive squares of the sequence share a side.
Prove that at least $\left(n^{2}-2\right) / 3$ checkers have been placed on the board.

## Mathematics Terms

CLEARLY: I don't want to write down all the "in- between" steps.
TRIVIAL: If I have to show you how to do this, you're in the wrong class.

## Calculus!

They chose an $\mathcal{E}$ that was so small that $\mathcal{E}^{2}$ was negative.
"The future science of government should be called 'la cybernetique'."

Andrè Marie AMPERE
[Asked for a testimony to the fact that Emmy Noether was a great woman mathematician, he said:] "I can testify that she is a great mathematician, but that she is a woman, I cannot swear."

Edmund LANDAU
"Physics is becoming too difficult for the physicists."

David HILBERT
"Father of Chemistry and Uncle of the Earl of Cork."

Robert BOYLE [On his tombstone]
"What I tell you three times is true."
Charles Lutwidge DOGSON
"If you are afraid of something, measure it, and you will realize it is a mere triple."

Renato CACCIOPPOLI


Rudi Mathematici
February

|  | 1 | S | (1900) John Charles BURKILL |
| :---: | :---: | :---: | :---: |
| 6 | 2 | M T | (1522) Lodovico FERRARI |
|  | 3 |  | (1893) Gaston Maurice JULIA |
|  | 4 | $\begin{aligned} & \mathrm{T} \\ & \mathrm{~W} \end{aligned}$ | (1905) Eric Cristopher ZEEMAN |
|  | 5 | W T | (1757) Jean Marie Constant DUHAMEL |
|  | 6 | F | (1612) Antoine ARNAULD <br> (1695) Nicolaus (II) BERNOULLI |
|  | 7 | S | (1877) Godfried Harold HARDY <br> (1883) Eric Temple BELL |
|  | 8 | S | (1700) Daniel BERNOULLI (1875) Francis Ysidro EDGEWORTH |
| 7 | 9 | M | (1775) Farkas Wolfgang BOLYAI (1907) Harod Scott MacDonald COXETER |
|  | $\begin{aligned} & 10 \\ & 11 \end{aligned}$ | $\begin{aligned} & \mathrm{T} \\ & \mathrm{~W} \end{aligned}$ | (1747) Aida YASUAKI |
|  |  |  | (1800) William Henry Fox TALBOT <br> (1839) Josiah Willard GIBBS <br> (1915) Richard Wesley HAMMING |
|  | 12 | T | (1914) Hanna CAEMMERER NEUMANN |
|  | 13 | F | (1805) Johann Peter Gustav Lejeune DIRICHLET |
|  | 14 | S | (1468) Johann WERNER (1849) Hermann HANKEL (1896) Edward Artur MILNE |
|  | 15 | S | $\begin{aligned} & \text { (1564) Galileo GALILEI } \\ & \text { (1861) Alfred North WHITEHEAD } \\ & \text { (1946) Douglas HOFSTADTER } \\ & \hline \end{aligned}$ |
| 8 | 16 | M | (1822) Francis GALTON <br> (1853) Georgorio RICCI-CURBASTRO <br> (1903) Beniamino SEGRE |
| 9 | 17 | T | (1890) Sir Ronald Aymler FISHER <br> (1891) Adolf Abraham Halevi FRAENKEL |
|  | $\begin{array}{ll} 18 & \mathrm{~W} \\ 19 & \mathrm{~T} \\ 20 & \mathrm{~F} \\ \mathbf{2 1} & \mathrm{~S} \end{array}$ |  | (1404) Leon Battista ALBERTI |
|  |  |  | (1473) Nicolaus COPERNICUS |
|  |  |  | (1844) Ludwig BOLTZMANN |
|  |  |  | (1591) Girard DESARGUES <br> (1915) Evgenni Michailovitch LIFSHITZ |
|  |  | $\begin{aligned} & \mathbf{S} \\ & \mathrm{M} \end{aligned}$ | (1903) Frank Plumpton RAMSEY |
|  |  |  | (1583) Jean-Baptiste MORIN <br> (1951) Shigefumi MORI |
|  | 24 | $\begin{aligned} & \hline \mathrm{T} \\ & \mathrm{~W} \\ & \mathrm{~T} \\ & \mathrm{~F} \\ & \mathrm{~S} \end{aligned}$ | (1871) Felix BERNSTEIN |
|  | 25 |  | (1827) Henry WATSON |
|  | 26 |  | (1786) Dominique Francois Jean ARAGO |
|  | 27 |  | (1881) Luitzen Egbertus Jan BROUWER |
|  | 28 |  | (1735) Alexandre Theophile VANDERMONDE |
|  |  |  | (1860) Herman HOLLERITH |

## USAMO 1999 - Pr. 2

Let $A B C D$ be a cyclic quadrilateral. Prove that
$|A B-C D|+|A D-B C| \geq 2|A C-B D|$.

## Mathematic Terms

OBVIOUSLY: I hope you weren't sleeping when we discussed this earlier, because I refuse to repeat it.
RECALL: I shouldn't have to tell you this, but for those of you who erase your memory tapes after every test...

## Mathematical Psychology

Zenophobia: the irrational fear of convergent sequences.
"Common sense is not really so common."
Antoine ARNAULD
"Technical skill is mastery of complexity while creativity is mastery of simplicity."

Eric Cristopher ZEEMAN
"Reductio ad absurdum, which Euclid loved so much, is one of a mathematician's finest weapons. It is a far finer gambit than any chess play: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers the game."

Godfried HARDY
"It would be better for the true physics if there were no mathematicians on earth."

Daniel BERNOULLI
"Epur si muove!"
Galileo GALILEI
"Connaitre, decouvrir, communiquer... telle est la destineè d'un savant."

Dominique ARAGO
"A mathematician will recognize Cauchy, Jacobi or Helmholtz after reading a few pages, just as a musician recognize, from the first few bars, Mozart, Beethoven or Schubert."

Ludwig BOLTZMANN
"Whenever you can, count."
Francis GALTON


## Rudi Mathematici

## March

|  | 1 | S | (1611) John PELL |
| :---: | :---: | :---: | :---: |
| 10 | 2 | M | (1836) Julius WEINGARTEN |
|  | 3 | T | (1838) George William HILL (1845) Georg CANTOR |
|  | 4 | W | (1822) Jules Antoine LISSAJUS |
|  | 5 | T | (1512) Gerardus MERCATOR (1759) Benjamin GOMPERTZ (1817) Angelo GENOCCHI |
|  | 6 | F | (1866) Ettore BORTOLOTTI |
|  | 7 | S | (1792) William HERSCHEL (1824) Delfino CODAZZI |
|  | 8 | S | (1851) George CHRYSTAL |
| 11 | 9 | M | (1818) Ferdinand JOACHIMSTHAL (1900) Howard Hathaway AIKEN |
|  | 10 | T | (1864) William Fogg OSGOOD |
|  | 11 | W | (1811) Urbain Jean Joseph LE VERRIER (1853) Salvatore PINCHERLE |
|  | 12 | T | (1685) George BERKELEY (1824) Gustav Robert KIRKHHOFF <br> (1859) Ernesto CESARO |
|  | 13 | F | (1861) Jules Joseph DRACH (1957) Rudy D'ALEMBERT |
|  | 14 | S | (1864) Jozef KURSCHAK (1879) Albert EINSTEIN |
|  | 15 | S | (1860) Walter Frank Raphael WELDON (1868) Grace CHISOLM YOUNG |
| 12 | 16 | M | (1750) Caroline HERSCHEL <br> (1789) Georg Simon OHM <br> (1846) Magnus Gosta MITTAG-LEFFLER |
|  | $17$ | T | (1876) Ernest Benjamin ESCLANGON (1897) Charles FOX |
|  | 18 | W | (1640) Philippe de LA HIRE (1690) Christian GOLDBACH (1796) Jacob STEINER |
|  | 19 | T | (1862) Adolf KNESER (1910) Jacob WOLFOWITZ |
|  | 20 | F | (1840) Franz MERTENS (1884) Philip FRANCK (1938) Sergi Petrovich NOVIKOV |
|  | 21 | S | (1768) Jean Baptiste Joseph FOURIER (1884) George David BIRKHOFF |
|  | 22 | S | (1917) Irving KAPLANSKY |
| 13 | 23 | M | (1754) Georg Freiherr von VEGA (1882) Emmy Amalie NOETHER (1897) John Lighton SYNGE |
|  | 24 | T | (1809) Joseph LIOUVILLE (1948) Sun-Yung (Alice) CHANG |
|  | 25 | W | (1538) Christopher CLAUSIUS |
|  | 26 | T | (1848) Konstantin ADREEV (1913) Paul ERDOS |
|  | 27 | F | (1857) Karl PEARSON |
|  | 28 | S | (1749) Pierre Simon de LAPLACE |
|  | 29 | S | (1825) Francesco FAȦ DI BRUNO (1873) Tullio LEVI-CIVITA (1896) Wilhelm ACKERMAN |
| 14 | 30 | M | (1892) Stefan BANACH |
|  | 31 | T | (1596) René DESCARTES |

## USAMO 1999 - Pr. 3

Let $p>2$ be a prime and let $a, b, c, d$ be integers not divisible by $p$, such that
$\left\{\frac{r a}{p}\right\}+\left\{\frac{r b}{p}\right\}+\left\{\frac{r c}{p}\right\}+\left\{\frac{r d}{p}\right\}=2$
for any integer $r$ not divisible by $p$. Prove that at least two of the numbers $a+b, a+c$, $a+d, b+c, b+d, c+d$
are divisible by $p$.
(Note: $\{x\}=x-\lfloor x\rfloor$ denotes the fractional part of $x$.)

## Mathematic Terms

WLOG (Without Loss Of Generality): I'm not about to do all the possible cases, so I'll do one and let you figure out the rest.
IT CAN EASILY BE SHOWN: Even you, in your finite wisdom, should be able to prove this without me holding your hand.

## The Four Operations

Ambition, distraction, uglification and derision. (L. Carroll)
"Common sense is nothing more than a deposit of prejudices laid down in the mind before you reach eighteen."

## Albert EINSTEIN

"We [he and Halmos] share a philosophy about linear algebra: we think basis-free, we write basis-free, but when the chips are down we close the office door and compute with matrices like fury."

Irving KAPLANSKY
"A Mathematician is a machine for turning coffee into theorems."

Paul ERDOS
"Perfect numbers (like perfect men) are very rare."

René DESCARTES
"A mathematician is a person who can find analogies between theorems; a better mathematician is one who can see analogies between proofs and the best mathematician can notice analogies between theories. One can imagine that the ultimate mathematician is one who can see analogies between analogies."

Stefan BANACH


|  | 1 | W | (1640) Georg MOHR (1776) Marie-Sophie GERMAIN (1895) Alexander Craig AITKEN |
| :---: | :---: | :---: | :---: |
|  |  | T | (1934) Paul Joseph COHEN |
|  | 3 | F | (1835) John Howard Van AMRINGE <br> (1892) Hans RADEMACHER <br> (1900) Albert Edward INGHAM <br> (1909) Stanislaw Marcin ULAM <br> (1971) Alice RIDDLE |
|  |  | S | (1809) Benjamin PEIRCE <br> (1842) Francois Edouard Anatole LUCAS <br> (1949) Shing-Tung YAU |
|  | 5 | S | (1588) Thomas HOBBES <br> (1607) Honorè FABRI <br> (1622) Vincenzo VIVIANI <br> (1869) Sergi Alexeievich CHAPLYGIN |
| 15 | 6 | M | (1801) William Hallowes MILLER |
|  | 7 | T | (1768) Francais Joseph FRANCAIS |
|  | 8 | W | (1903) Marshall Harvey STONE |
|  | 9 | T | (1791) George PEACOCK (1816) Charles Eugene DELAUNAY (1919) John Presper HECKERT |
|  | 10 | F | (1857) Henry Ernest DUDENEY |
|  | 11 | S | (1953) Andrew John WILES |
|  | 12 | S | (1794) Germinal Pierre DANDELIN <br> (1852) Carl Louis Ferdinand Von LINDEMANN <br> (1903) Jan TINBERGEN |
| 16 | 13 | M | (1728) Paolo FRISI <br> (1813) Duncan Farquharson GREGORY <br> (1879) Francesco SEVERI |
|  | 14 | T | (1629) Christiaan HUYGENS |
|  | 15 | W | (1452) Leonardo da VINCI <br> (1548) Pietro Antonio CATALDI <br> (1707) Leonhard EULER <br> (1809) Herman Gunther GRASSMANN |
|  |  | T | (1682) John HADLEY <br> (1823) Ferdinand Gotthold Max EISENSTEIN |
|  |  | F | (1798) Etienne BOBILLIER <br> (1853) Arthur Moritz SCHONFLIES |
|  | $18$ | S | (1907) Lars Valerian AHLFORS <br> (1918) Hsien Chung WANG <br> (1949) Charles Luois FEFFERMAN |
|  | 19 | S | (1880) Evgeny Evgenievich SLUTSKY <br> (1883) Richard VIN MISES <br> (1901) Kiyoshi OKA <br> (1905) Charles EHRESMANN <br> $(839)$ |
| 17 | 20 | M | (1839) Francesco SIACCI |
|  | $21$ | T | (1652) Michel ROLLE <br> (1774) Jean Baptiste BIOT <br> (1875) Teiji TAKAGI |
|  | 22 | W | (1811) Otto Ludwig HESSE <br> (1887) Harald August BOHR |
|  | 23 | T | (1858) Max Karl Ernst Ludwig PLANCK |
|  | 24 | F | (1863) Giovanni VAILATI |
|  | $25$ | S | (1849) Felix Christian KLEIN <br> (1900) Wolfgang PAULI <br> (1903) Andrei Nicolayevich KOLMOGOROV |
|  | 26 | S | (1889) Ludwig Josef Johan WITTENGSTEIN |
| 18 | 27 | M | (1755) Marc-Antoine PARSEVAL des Chenes |
|  | 28 | T | (1906) Kurt GODEL |
|  | 29 | W | (1854) Jules Henri POINCARÈ |
|  | 30 | T | (1777) Johann Carl Friedrich GAUSS (1916) Claude Elwood SHANNON |

## USAMO 1999 - Pr. 4

Let $a_{1}, a_{2}, \ldots, a_{n}(n>3)$ be real numbers such that:

$$
a_{1}+a_{2}+\ldots+a_{n} \geq n
$$

And

$$
a_{1}^{2}+a_{2}^{2}+\ldots+a_{n}^{2} \geq n^{2}
$$

Prove that $\max \left(a_{1}, a_{2}, \ldots, a_{n}\right) \geq 2$.

## Mathematic Terms

CHECK (or CHECK FOR YOURSELF): This is the boring part of the proof, so you can do it on your own time.
SKETCH OF A PROOF: I couldn't verify all the details, so I'll break it down into the parts I couldn't prove.

## Formally Correct

Q: What's the difference between the radius and the diameter of a circle?
A: The radius.
"Knowing what is big and what is small is more important than being able to solve partial differential equations"

Stanislaw Marcin ULAM
"You treat world history as a mathematician does mathematics, in which nothing but laws and formulae exist, no reality, no good and evil, no time, no yesterday, no tomorrow, nothing but an eternal shallow, mathematical present."

Otto Ludwig HESSE
" The fact that the author thinks slowly is not serious, but the fact that he publishes faster than he thinks is inexcusable."

Wolfgang PAULI
"Everyone knows what a curve is, until he has studied enough mathematics to become confused through the countless number of possible exceptions."

Felix KLEIN


## Rudi Mathematici

|  |  | F | (1825) Johann Jacob BALMER |
| :---: | :---: | :---: | :---: |
|  | 2 | S | (1860) D'Arcy Wentworth THOMPSON (1905) Kazimierz ZARANKIEWITZ |
|  | 3 | S | (1842) Otto STOLZ <br> (1860) Vito VOLTERRA |
| 19 | $\begin{array}{ll} \hline \mathbf{4} & \mathrm{M} \\ \mathbf{5} & \mathrm{~T} \\ \mathbf{6} & \mathrm{~W} \\ \mathbf{7} & \mathrm{~T} \end{array}$ |  | (1845) William Kingdon CLIFFORD |
|  |  |  | (1833) Lazarus Emmanuel FUCHS (1897) Francesco Giacomo TRICOMI |
|  |  |  | (1872) Willem DE SITTER (1906) Andrè VEIL |
|  |  |  | (1926) Alexis Claude CLAIRAUT <br> (1854) Giuseppe VERONESE <br> (1881) Ebenezer CUNNINGHAM <br> (1896) Pavel Sergieievich ALEXANDROV |
|  | $\begin{gathered} 8 \\ 9 \\ 10 \end{gathered}$ |  | (1859) Johan Ludwig William Valdemar JENSEN |
|  |  | $\mathbf{S}$ | (1746) Gaspard MONGE <br> (1876) Gilbert Ames BLISS |
|  |  |  | (1788) Augustin Jean FRESNEL <br> (1847) William Karl Joseph KILLING <br> (1958) Piotr Rezierovic SILVERBRAHMS |
| 20 | $\begin{array}{ll} \hline 11 & M \\ 12 & T \end{array}$ |  | (1918) Richard Phillips FEYNMAN |
|  |  |  | (1845) Pierre RenéJean Baptiste Henry BROCARD (1902) Frank YATES |
|  | $\begin{array}{ll} 13 & \mathrm{~W} \\ 14 & \mathrm{~T} \end{array}$ |  | (1750) Lorenzo MASCHERONI |
|  |  |  | (1832) Rudolf Otto Sigismund LIPSCHITZ <br> (1863) John Charles FIELDS |
|  | 15 | F | (1939) Brian HARTLEY |
|  | 16 | S | (1718) Maria Gaetana AGNESI <br> (1821) Pafnuti Lvovi CHEBYSHEV |
|  | 17 | S | (1940) Alan KAY |
| 21 | $\begin{aligned} & 18 \\ & 19 \\ & 20 \\ & 21 \\ & 22 \\ & 23 \\ & 24 \\ & \hline \end{aligned}$ | $\bar{M}$ | (1850) Oliver HEAVISIDE <br> (1892) Bertrand Arthur William RUSSELL |
|  |  | $\mathrm{T}$ | (1919) Georgii Dimitirievich SUVOROV |
|  |  | W | (1861) Henry Seely WHITE |
|  |  | $\mathrm{T}$ | (1471) Albrecht DURER <br> (1792) Gustave Gaspard de CORIOLIS |
|  |  | F | (1865) Alfred Cardew DIXON |
|  |  | S | (1914) Lipa BERS |
|  |  | S | (1544) William GILBERT |
| 22 | 25 M <br> 26 T |  | (1838) Karl Mikailovich PETERSON |
|  |  |  | (1667) Abraham DE MOIVRE <br> (1896) Yuri Dimitrievich SOKOLOV |
|  | $\begin{array}{ll} 27 & \text { W } \\ 28 & \text { T } \end{array}$ |  | (1862) John Edward CAMPBELL |
|  |  |  | (1676) Jacopo Francesco RICCATI (1710) Johann (II) BERNOULLI |
|  | $\begin{array}{ll} 29 & \mathrm{~F} \\ 30 & \mathrm{~S} \\ 31 & \mathrm{~S} \\ \hline \end{array}$ |  | (1882) Harry BATEMAN |
|  |  |  | (1814) Eugene Charles CATALAN |
|  |  |  | (1926) John KEMENY |

## USAMO, 1999 - Pr. 5

The Y2K Game is played on a $1 \times 2000$ grid as follows. Two players in turn write either an S or an O in an empty square. The first player who produces three consecutive boxes that spell SOS wins. If all boxes are filled without producing SOS then the game is a draw.
Prove that the second player has a winning strategy.

## Mathematic Terms

HINT: The hardest of several possible ways to do a proof.
BRUTE FORCE: Four special cases, three counting arguments, two long inductions, "and a partridge in a pair tree."

## Back to Reality

Q: What is the physicist's definition of a vector space?
A: A set V satisfying the axiom that for any $x$ in $\mathrm{V}, x$ has a little arrow drawn over it.
"Nature is not embarrassed by difficulties of analysis."

Augustin Jean FRESNEL
"To those who do not knows mathematics it is difficult to get across a real feeling as to the deepest beauty of nature [...] If you want to appreciate nature, it is necessary to understand the language that she speaks in."

Richard Phillips FEYNMAN
"To isolate mathematics from the practical demands of the sciences is to invite the sterility of a cow shut away from the bulls."

Pafnuti Lvovi CHEBYSHEV
"Mathematics is very much like poetry. What makes a great poem is that there is a great amount of thought expressed in very few words. In this sense, formulas like $\mathrm{e}^{\pi_{i}+1}=0$ are poems."

Lipa BERS
"This series is divergent, therefore we may be able to do something with it."

Oliver HEAVISIDE
"Men who are unhappy, like men who sleep badly, are always proud of the fact."

Bertrand RUSSELL
"A quantity that is increased or decreased by an infinitely small quantity is neither increased nor decreased."

Johann BERNOULLI


## Rudi Mathematici

| 23 | 1 | M | (1796) Sadi Leonard Nicolas CARNOT <br> (1851) Edward Bailey ELLIOTT <br> (1899) Edward Charles TITCHMARSH |
| :---: | :---: | :---: | :---: |
|  | 2 | T | (1895) Tibor RADÒ |
|  | 3 | W | (1659) David GREGORY |
|  | 4 | T | (1809) John Henry PRATT |
|  | 5 | F | (1814) Pierre LAurent WANTZEL (1819) John Couch ADAMS |
|  | 6 | S | (1436) Johann Muller REGIOMONTANUS (1857) Aleksandr Michailovitch LYAPUNOV (1906) Max ZORN |
|  | 7 | S | (1863) Edward Burr VAN VLECK |
| 24 | 8 | M | (1625) Giovanni Domenico CASSINI <br> (1858) Charlotte Angas SCOTT <br> (1860) Alicia Boole STOTT |
|  | 9 | T | (1885) John Edensor LITTLEWOOD |
|  | 10 | W | (940) Mohammad ABU'L WAFA Al-Buzjani (1887) Vladimir Ivanovich SMIRNOV |
|  | 11 | T | (1937) David Bryant MUMFORD |
|  | 12 | F | (1888) Zygmunt JANYSZEWSKI |
|  | 13 | S | (1831) James Clerk MAXWELL <br> (1876) William Sealey GOSSET (Student) <br> (1928) John Forbes NASH |
|  | 14 | S | (1736) Charles Augustin de COULOMB (1856) Andrei Andreyevich MARKOV (1903) Alonzo CHURCH |
| 25 | 15 | M | (1640) Bernard LAMY <br> (1894) Nikolai Gregorievich CHEBOTARYOV |
|  | 16 | T | (1915) John Wilder TUKEY |
|  | 17 | W | (1898) Maurits Cornelius ESCHER |
|  | 18 | T | (1858) Andrew Russell FORSYTH <br> (1884) Charles Ernest WEATHERBURN |
|  | 19 | F | (1623) Blaise PASCAL <br> (1902) Wallace John ECKERT |
|  | 20 | S | (1873) Alfred LOEWY |
|  | 21 | S | (1781) Simeon Denis POISSON (1828) Giuseppe BRUNO |
| 26 | 22 | M | (1823) Mario PIERI <br> (1864) Hermann MINKOWSKY <br> (1910) Konrad ZUSE |
|  | 23 | T | (1912) Alan Mathison TURING |
|  | 24 | W | (1880) Oswald VEBLEN |
|  | 25 | T | (1908) William Van Orman QUINE |
|  | 26 | F | (1824) William THOMPSON, Lord Kelvin (1918) Yudell Leo LUKE |
|  | 27 | S | (1806) Augustus DE MORGAN |
|  | 28 | S | (1875) Henri Leon LEBESGUE |
| 27 | 29 | M | (1888) Aleksandr Aleksandrovich FRIEDMANN |
|  | 30 | T | (1791) Felix SAVART |

## USAMO 1999 - Pr. 6

Let $A B C D$ be an isosceles trapezoid with $A B \| C D$. The inscribed circle $\omega$ of triangle $B C D$ meets $C D$ at $E$. Let $F$ be a point on the (internal) angle bisector of $D \hat{A C}$ such that $E F \perp C D$. Let the circumscribed circle of triangle $A C F$ meet line $C D$ at $C$ and $G$.
Prove that the triangle $A F G$ is isosceles.

## Mathematic Terms

SOFT PROOF: One third less filling (of the page) than your regular proof, but it requires two extra years of course work just to understand the terms.
ELEGANT PROOF: Requires no previous knowledge of the subject matter and is less than ten lines long.

## Topology

A topologist is a man who doesn't know the difference between a coffee up and a doughnut.
"It can be of no practical use to know that $\pi$ is irrational, but if we can know, it surely would be intolerable not to know."

Edward Charles TITCHMARSH
"What I give form to in daylight is only one per cent of what I have seen in darkness."

Maurits Cornelius ESCHER
"The more I see of men, the better I like my dog."

Blaise PASCAL
"Science is a differential equation. Religion is a boundary condition."

Alan Mathison TURING
"In my opinion, a mathematician, in so far as he is a mathematician, need not preoccupy himself with philosophy - an opinion, moreover, which has been expressed by many philosophers."

## Henri LEBESGUE

"Try a hard problem. You may not solve it, but you will prove something else."
"Mathematics is a dangerous profession; an appreciable proportion of us goes mad."

John LITTLEWOOD
"The mathematical education of the young physicist [A. Einstein] was not very solid, which I am in good position to evaluate since he obtained it from me in Zurich some time ago."

Hermann MINKOWSKI


## Rudi Mathematici

|  | 1 | W | (1643) Gottfried Wilhelm von LEIBNIZ (1788) Jean Victor PONCELET |
| :---: | :---: | :---: | :---: |
|  | 2 | T | (1820) William John Racquorn RANKINE (1852) William BURNSIDE |
|  | 3 | F | (1807) Ernest Jean Philippe Fauque de JONQUIERE (1897) Jesse DOUGLAS |
|  | 4 | S | (1906) Daniel Edwin RUTHERFORD (1917) Michail Samuilovich LIVSIC |
|  | 5 | S | (1936) James MIRRLEES |
| 28 | 6 | M | (1849) Alfred Bray KEMPE |
|  | 7 | T | (1816) Johann Rudolf WOLF <br> (1906) William FELLER <br> (1922) Vladimir Aleksandrovich MARCHENKO |
|  | 8 | W | (1760) Christian KRAMP |
|  | 9 | T | (1845) George Howard DARWIN |
|  | 10 | F | (1862) Roger COTES <br> (1868) Oliver Dimon KELLOGG |
|  | 11 | S | (1857) Sir Joseph LARMOR (1890) Giacomo ALBANESE |
|  | 12 | S | (1875) Ernest Sigismund FISCHER <br> (1895) Richard BUCKMINSTER FULLER |
| 29 | 13 | M | (1527) John DEE <br> (1741) Karl Friedrich HINDENBURG |
|  | 14 | T | (1671) Jacques D'ALLONVILLE (1793) George GREEN |
|  | 15 | W | (1865) Wilhelm WIRTINGER (1906) Adolph Andrej Pavlovich YUSHKEVICH |
|  | 16 | T | (1678) Jakob HERMANN <br> (1903) Irmgard FLUGGE-LOTZ |
|  | 17 | F | (1831) Victor Mayer Amedeè MANNHEIM (1837) Wilhelm LEXIS |
|  | 18 | S | (1013) Hermann von REICHENAU (1635) Robert HOOKE (1853) Hendrich Antoon LORENTZ |
|  | 19 | S | (1768) Francois Joseph SERVOIS |
| 30 | 20 | M | (1876) Otto BLUMENTHAL (1947) Gerd BINNIG |
|  | 21 | T | (1620) Jean PICARD (1848) Emil WEYR (1849) Robert Simpson WOODWARD |
|  | 22 | W | (1784) Friedrich Wilhelm BESSEL |
|  | $23$ | T | (1775) Etienne Louis MALUS (1854) Ivan SLEZYNSKY |
|  | 24 | F | (1851) Friedrich Herman SCHOTTKY (1871) Paul EPSTEIN (1923) Christine Mary HAMILL |
|  | 25 | S | (1808) Johann Benedict LISTING |
|  | 26 | S | (1903) Kurt MAHLER |
| 31 | 27 | M | (1667) Johann BERNOULLI <br> (1801) George Biddel AIRY <br> (1848) Lorand Baron von EOTVOS <br> (1871) Ernst Friedrich Ferdinand ZERMELO |
|  | 28 | T | (1954) Gerd FALTINGS |
|  | 29 | W | (1898) Isidor Isaac RABI |
|  | 30 | T | (1889) Vladimir Kosma ZWORKYN |
|  | 31 | F | (1704) Gabriel CRAMER <br> (1712) Johann Samuel KOENIG |

## USAMO 2000, Pr. 1

Call a real-value function $f$ very convex if:

$$
\frac{f(x)+f(y)}{2} \geq f\left(\frac{x+y}{2}\right)+|x+y|
$$

holds for all real numbers $x$ and $y$. Prove that no very convex function exists.

## Mathematics Terms

SIMILARLY: At least one line of the proof of this case is the same as before.
CANONICAL FORM: 4 out of 5 mathematicians surveyed recommended this as the final form for their students who choose to finish.

## Algebra

In modern mathematics, algebra has become so important that numbers will soon only have symbolic meaning.
"When working on a problem, I never think about beauty; I think only of how to solve the problem. But when I have finished, if the solution is not beautiful, I know that it is wrong."

## Richard Buckminster FULLER

" There is (gentle reader) nothing (the works of God only set apart) which so much beautifies and adorns the soul and mind of man as does knowledge of the good arts and sciences. ... Many ... arts there are which beautify the mind of man; but of all none do more garnish and beautify it than those arts which are called mathematical, unto the knowledge of which no man can attain, without perfect knowledge and instruction of the principles, grounds, and Elements of Geometry."

John DEE

## "CEIIOSSOTTUU"

Anagram to establish priority in the discovery of elasticity: "Ut tensio, sic uis".

Robert HOOKE
"Miracles are not to be multiplied beyond necessity."
"Taking mathematics from the beginning of the word to the time of Newton, what he has done is much the better half."

Gottfried LEIBNIZ
"[The infinitesimals] neither have nor can have theory: in practise it is a dangerous instrument in the hands of beginners [...] anticipating, for my part, the judgement of posterity, I would predict that this method will be accused one day, and rightly, of having retarded the progress of the mathematical sciences."

Francois SERVOIS


## Rudi Mathematici

## August

|  |  |  | (1861) Ivar Otto BENDIXSON <br> (1881) Otto TOEPLITZ |
| :---: | :---: | :---: | :---: |
|  | 2 | S | (1856) Ferdinand RUDIO <br> (1902) Mina Spiegel REES |
| 32 | 3 | M | (1914) Mark KAC |
|  | 4 | T | (1805) Sir William Rowan HAMILTON (1838) John VENN |
|  | 5 | W | (1802) Niels Henrik ABEL |
|  | 6 | T | (1638) Nicolas MALEBRANCHE <br> (1741) John WILSON |
|  | 7 | F | (1868) Ladislaus Josephowitsch BORTKIEWITZ |
|  | 8 | S | (1902) Paul Adrien Maurice DIRAC |
|  | 9 | S | (1537) Francesco BAROZZI (Franciscus Barocius) |
| 33 | 10 | M | (1602) Gilles Personne de ROBERVAL |
|  | 11 | T | (1730) Charles BOSSUT <br> (1842) Enrico D'OVIDIO |
|  | 12 | W | (1882) Jules Antoine RICHARD <br> (1887) Erwin Rudolf Josef Alexander SCHRODINGER |
|  | 13 | T | (1625) Erasmus BARTHOLIN (1819) George Gabriel STOKES (1861) Cesare BURALI-FORTI |
|  |  | F | (1530) Giovanni Battista BENEDETTI <br> (1842) Jean Gaston DARBOUX <br> (1865) Guido CASTELNUOVO <br> (1866) Charles Gustave Nicolas de la VALLEÈ POUSSIN |
|  |  | S | (1863) Aleksei Nikolaevich KRYLOV <br> (1892) Louis Pierre Victor duc de BROGLIE <br> (1901) Petr Sergeevich NOVIKOV |
|  | 16 | S | (12773) Louis Beniamin FRANCOEUR <br> (1821) Arthur CAYLEY |
| 34 | 17 | M | (1601) Pierre de FERMAT |
|  | 18 | T | (1685) Brook TAYLOR |
|  | $19$ | W | (1646) John FLAMSTEED <br> (1739) Georg Simon KLUGEL |
|  | 20 | T | (1710) Thomas SIMPSON (1863) Corrado SEGRE (1882) Waclav SIERPINSKI |
|  | 21 | F | (1789) Augustin Louis CAUCHY |
|  | 22 | S | (1647) Denis PAPIN |
|  | 23 | S | (1683) Giovanni POLENI <br> (1829) Moritz Benedikt CANTOR |
| 35 |  | M | (1561) Bartholomeo PITISCUS <br> (1942) Karen Keskulla UHLENBECK |
|  | $25$ | $\mathrm{T}$ | (1561) Philip van LANSBERGE <br> (1844) Thomas MUIR |
|  |  | W | (1728) Johann Heinrich LAMBERT (1875) Giuseppe VITALI |
|  | 27 | T | (1858) Giuseppe PEANO |
|  | 28 | F | (1796) Irenee Jules BIENAYMÈ |
|  | 29 | S | (1904) Leonard ROTH |
|  |  | S | (1856) Carle David Tolmè RUNGE (1906) Olga TAUSSKY-TODD |
| 36 | 31 | M | (1821) Hermann Ludwig Ferdinand von HELMHOLTZ |

## USAMO 2000, Pr. 2

Let $S$ be the set of all triangles $A B C$ for which:

$$
5\left(\frac{1}{A P}+\frac{1}{B Q}+\frac{1}{C R}\right)-\frac{3}{\min \{A P, B Q, C R\}}=\frac{6}{r}
$$

Where $r$ is the inradius and $P, Q, R$ are the points of tangency of the incircle with sides $A B, B C, C A$ respectively.
Prove that all triangles in $S$ are isosceles and similar to one another.

## Mathematic Terms

TFAE (The Following Are Equivalent): If I say this it means that, and if I say that it means the other thing, and if I say the other thing...
BY A PREVIOUS THEOREM: I don't remember how it goes (come to think of it I'm not really sure we did this at all), but if I stated it right (or at all), then the rest of this follows.

## Prime Numbers

It was mentioned on CNN that the new prime number discovered recently is four times bigger than the previous record.
"This result is too beautiful to be false; it is more important to have beauty in one's equations than to have them fit experiment."

Paul Adrien Maurice DIRAC
"And perhaps, posterity will thank me for having shown it that the ancients did not know everything."

Pierre de FERMAT
"Cubum autem in duos cubos, aut quadrato quadratum in duos quadrato quadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos ejusdem nominis fas est dividere: cujus rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet."

Pierre de FERMAT
"As for everything else, so for a mathematical theory: beauty can be perceived but not explained."

Arthur CAYLEY
"There are surely worse things than being wrong, and being dull and pedantic are surely among them."

Mark KAC
"Whoever [in the pursuit of science] seeks after immediate practical utility may rest assured that he seeks in vain."

Hermann von HELMHOLTZ


## Rudi Mathematici

 September|  |  | T | (1659) Joseph SAURIN <br> (1835) William Stankey JEVONS |
| :---: | :---: | :---: | :---: |
|  | 2 | W | (1878) Mauriche René FRECHET <br> (1923) René THOM |
|  | 3 | T | (1814) James Joseph SYLVESTER <br> (1884) Solomon LEFSCHETZ <br> (1908) Lev Semenovich PONTRYAGIN |
|  | 4 | F | (1809) Luigi Federico MENABREA |
|  | 5 | S | (1667) Giovanni Girolamo SACCHERI <br> (1725) Jean Etienne MONTUCLA |
|  | 6 | S | (1859) Boris Jakovlevich BUKREEV <br> (1863) Dimitri Aleksandrovich GRAVE |
| 37 | 7 | M | (1707) George Louis Leclerc comte de BUFFON (1955) Efim ZELMANOV |
|  | 8 | T | (1584) Gregorius SAINT-VINCENT (1588) Marin MERSENNE |
|  | 9 | W | (1860) Frank MORLEY |
|  | 10 | T | (1839) Charles Sanders PEIRCE |
|  |  | F | (1623) Stefano degli ANGELI <br> (1877) sir James Hopwood JEANS |
|  |  | S | (1891) Antoine Andrè Louis REYNAUD (1900) Haskell Brooks CURRY |
|  | 13 | S | (1873) Constantin CARATHEODORY <br> (1885) Wilhelm Johann Eugen BLASCHKE |
| 38 |  |  | (1858) Henry Burchard FINE <br> (1891) Ivan Matveevich VINOGRADOV |
|  | 15 | T | (973) Abu Arrayhan Muhammad ibn Ahmad AL'BIRUNI (1886) Paul Pierre LEVY |
|  | 16 | W | (1494) Francisco MAUROLICO <br> (1736) Johann Nikolaus TETENS |
|  | 17 | T | (1743) Marie Jean Antoine Nicolas de Caritat de CONDORCET (1826) Georg Friedrich Bernhard RIEMANN |
|  | 18 | F | (1752) Adrien Marie LEGENDRE |
|  | 19 | S | (1749) Jean Baptiste DELAMBRE |
|  | 20 | S | (1842) Alexander Wilhelm von BRILL <br> (1861) Frank Nelson COLE |
| 39 | $\begin{aligned} & 21 \\ & 22 \end{aligned}$ |  | (1899) Juliusz Pawel SCHAUDER |
|  |  | $\begin{aligned} & \mathrm{IV} \\ & \mathrm{~T} \end{aligned}$ | (1765) Paolo RUFFINI <br> (1769) Louis PUISSANT <br> (1803) Jaques Charles Francois STURM |
|  | 23 | W | (1768) William WALLACE (1900) David van DANTZIG |
|  | 24 | T | (1501) Girolamo CARDANO <br> (1625) Johan DE WITT <br> (1801) Michail Vasilevich OSTROGRADSKI |
|  | 25 | F | (1819) George SALMON <br> (1888) Stefan MAZURKIEWICZ |
|  | 26 | S | (1688) Willem Jakob ‘s GRAVESANDE (1854) Percy Alexander MACMAHON (1891) Hans REICHENBACH |
|  | 27 | S | (1855) Paul Emile APPEL <br> (1876) Earle Raymond HEDRICK <br> (1919) James Hardy WILKINSON |
| 40 | 28 | M | (1698) Pierre Louis Moreau de MAUPERTUIS <br> (1761) Ferdinand Francois Desirè Budan de BOISLAURENT <br> (1873) Julian Lowell COOLIDGE |
|  | 29 | T | (1561) Adriaan van ROOMEN (1812) Adolph GOPEL |
|  |  |  | (1775) Robert ADRAIN <br> (1829) Joseph WOLSTENHOLME <br> (1883) Ernst HELLINGER |


#### Abstract

USAMO 2000 - Pr. 3 A game of solitaire is played with $R$ red cards, $W$ white cards, and $B$ blue cards. A player plays all the cards one at a time. With each play he accumulates a penalty. If he plays a blue card, then he is charged a penalty which is the number of white cards still in his hand. If he plays a white card, then he is charged a penalty which is twice the number of red cards still in his hand. If he plays a red card, then he is charged a penalty which is three times the number of blue cards still in his hand.

Find, as a function of $R, W$ and $B$, the minimal total penalty a player can amass and all the ways in which this minimum can be achieved.


## Mathematic Terms

TWO LINE PROOF: I'll leave out everything but the conclusion, you can't question 'em if you can't see 'em.
BRIEFLY: I'm running out of time, so I'll just write and talk faster.

Pi
I've memorized 100,000 digits of pi. They're all 3. Of course I haven't memorized exactly where they occur.
"I believe that proving is not a natural activity for mathematicians."

René THOM
"If it's just turning the crank it's algebra, but if it's got an idea in it, it's topology."

Solomon LEFSCHETZ
"This branch of mathematics [Probability] is the only one, I believe, in which good writers frequently get results which are entirely erroneous."

Charles Sanders PEIRCE
"We may as well cut out the group theory. That is a subject that will never be of any use in physics."
sir James Hopwood JEANS
"If error is corrected whenever it is recognised, the path of error is the path of truth."

Hans REICHENBACH
"The early study of Euclid made me a hater of geometry."

James SYLVESTER
"If only I had the theorems! Then I should find the proofs easily enough..."

Bernhard RIEMANN


## Rudi Mathematici

## October

|  |  | T | (1671) Luigi Guido GRANDI (1898) Bela KEREKJARTÒ |
| :---: | :---: | :---: | :---: |
|  | 2 | F | (1825) John James WALKER <br> (1908) Arthur ERDELYI |
|  | 3 | S | (1944) Pierre René DELIGNE |
|  | 4 | S | (1759) Louis Francois Antoine ARBOGAST (1797) Jerome SAVARY |
| 41 | 5 | M | (1732) Nevil MASKELYNE <br> (1781) Bernhard Placidus Johann Nepomuk BOLZANO <br> (1861) Thomas Little HEATH |
|  | 6 | T | (1552) Matteo RICCI <br> (1831) Julius Wilhelm Richard DEDEKIND <br> (1908) Sergei Lvovich SOBOLEV |
|  | 789 | $\begin{aligned} & \mathrm{W} \\ & \mathrm{~T} \\ & \mathrm{~F} \end{aligned}$ | (1885) Niels BOHR |
|  |  |  | (1908) Hans Arnold HEILBRONN |
|  |  |  | (1581) Claude Gaspard BACHET de Meziriac (1704) Johann Andrea von SEGNER (1873) Karl SCHWARTZSCHILD |
|  | $\begin{aligned} & 10 \\ & 11 \end{aligned}$ | $\begin{aligned} & \mathrm{S} \\ & \mathrm{~S} \end{aligned}$ | (1861) Heinrich Friedrich Karl Ludwig BURKHARDT |
|  |  |  | (1675) Samuel CLARKE <br> (1777) Barnabè BRISSON <br> (1885) Alfred HAAR <br> (1910) Cahit ARF |
| 42 | $\begin{aligned} & 12 \\ & 13 \end{aligned}$ | $\begin{aligned} & \mathrm{M} \\ & \mathrm{~T} \end{aligned}$ | (1860) Elmer SPERRY |
|  |  |  | (1890) Georg FEIGL <br> (1893) Kurt Werner Friedrich REIDEMEISTER <br> (1932) John Griggs THOMSON |
|  | 14 | W | (1687) Robert SIMSON <br> (1801) Joseph Antoine Ferdinand PLATEAU <br> (1868) Alessandro PADOA |
|  | 15 | T | (1608) Evangelista TORRICELLI <br> (1735) Jesse RAMSDEN <br> (1776) Peter BARLOW |
|  | $\begin{aligned} & 16 \\ & 17 \end{aligned}$ | F | (1879) Philip Edward Bertrand JOURDAIN |
|  |  | S | (1759) Jacob (II) BERNOULLI (1888) Paul Isaac BERNAYS |
|  | 18 | S | (1741) John WILSON |
| 43 | 19 | M | (1903) Jean Frederic Auguste DELSARTE (1910) Subrahmanyan CHANDRASEKHAR |
|  | 20 | T | (1632) Sir Cristopher WREN <br> (1863) William Henry YOUNG <br> (1865) Aleksandr Petrovich KOTELNIKOV |
|  | 21 | W | (1677) Nicolaus (I) BERNOULLI <br> (1823) Enrico BETTI <br> (1855) Giovan Battista GUCCIA <br> (1893) William LEonard FERRAR |
|  | 22 | T | (1587) Joachim JUNGIUS <br> (1895) Rolf Herman NEVANLINNA <br> (1907) Sarvadaman CHOWLA |
|  | 23 | F | (1865) Piers BOHL |
|  | 24 | S | (1804) Wilhelm Eduard WEBER (1873) Edmund Taylor WITTAKER |
|  | 25 | S | (1811) Evariste GALOIS |
| 44 | 26 | M | (1849) Ferdinand Georg FROBENIUS <br> (1857) Charles Max MASON <br> (1911) Shiing-Shen CHERN |
|  | 27 | T | (1678) Pierre Remond de MONTMORT (1856) Ernest William HOBSON |
|  | 28 | W | (1804) Pierre Francois VERHULST |
|  | 29 |  | (1925) Klaus ROTH |
|  | 30 | F | (1906) Andrej Nikolaevich TIKHONOV |
|  | 31 | S | (1815) Karl Theodor Wilhelm WEIERSTRASS |


#### Abstract

USAMO 2000 - Pr. 4 Find the smallest positive integer $n$ such that if $n$ squares of a $1000 \times 1000$ chessboard are coloured, then there will exist three coloured squares whose centres form a right triangle with sides parallel to the edges of the board.


## Mathematic Terms

LET'S TALK THROUGH IT: I don't want to write it on the board lest I make a mistake.
PROCEED FORMALLY: Manipulate symbols by the rules without any hint of their true meaning (popular in pure math courses).

## Ancient Mathematics

The Romans didn't find algebra very challenging, because X was always 10 .
"An expert is a man who has made all the mistakes which can be made in a very narrow field."
"Anyone who is not shocked by quantum theory has not understood it."
"Prediction is very difficult, especially about the future."
"How wonderful that we have met with a paradox. Now we have some hope of making progress."

Niels BOHR
" $2^{30}\left(2^{31}-1\right)$ is the greatest perfect number that will ever be discovered, for, as they are merely curious without being useful, it is not likely that any person will attempt to find a number beyond it."

Peter BARLOW
"Unfortunately what is little recognized is that the most worthwhile scientific books are those in which the author clearly indicates what he does not know; for an author most hurts his readers by concealing difficulties."

Evariste GALOIS
"It is true that a mathematician who is not also something of a poet will never be a perfect mathematician."

Karl Theodor Wilhelm WEIERSTRASS


## November

|  | 1 | S | (1535) Giambattista DELLA PORTA |
| :---: | :---: | :---: | :---: |
| 45 | 2 M <br> 3 T |  | (1815) George BOOLE |
|  |  |  | (1867) Martin Wilhelm KUTTA (1878) Arthur Byron COBLE |
|  | 4 | W | (1744) Johann (III) BERNOULLI (1865) Pierre Simon GIRARD |
|  | 5 | T | (1848) James Whitbread Lee GLAISHER (1930) John Frank ADAMS |
|  | 6 | F | (1781) Giovanni Antonio Amedeo PLANA |
|  | 7 | S | (1660) Thomas Fantet DE LAGNY (1799) Karl Heinrich GRAFFE (1898) Raphael SALEM |
|  | 8 | S | (1656) Edmond HALLEY <br> (1846) Eugenio BERTINI <br> (1848) Fredrich Ludwig Gottlob FREGE <br> (1854) Johannes Robert RYDBERG <br> (1869) Felix HAUSDORFF |
| 46 | 9 | M | (1847) Carlo Alberto CASTIGLIANO <br> (1885) Theodor Franz Eduard KALUZA <br> (1885) Hermann Klaus Hugo WEYL <br> (1906) Jaroslav Borisovich LOPATYNSKY <br> (1922) Imre LAKATOS |
|  | $\begin{array}{ll} \mathbf{1 0} & \mathrm{T} \\ \mathbf{1 1} & \mathrm{~W} \\ \mathbf{1 2} & \mathrm{~T} \end{array}$ |  | (1829) Helwin Bruno CHRISTOFFEL |
|  |  |  | (1904) John Henry Constantine WHITEHEAD |
|  |  |  | (1825) Michail Egorovich VASHCHENKO-ZAKHARCHENKO (1842) John William STRUTT Lord RAYLEIGH (1927) Yutaka TANIYAMA |
|  | $\begin{array}{ll} 13 & \mathrm{~F} \\ 14 & \mathrm{~S} \\ 15 & \mathrm{~S} \end{array}$ |  | (1876) Ernest Julius WILKZYNSKY <br> (1878) Max Wilhelm DEHN |
|  |  |  | (1845) Ulisse DINI |
|  |  |  | (1688) Louis Bertrand CASTEL <br> (1793) Michel CHASLES <br> (1794) Franz Adolph TAURINUS |
| 47 | $\begin{array}{ll} \hline 16 & \mathrm{M} \\ 17 & \mathrm{~T} \end{array}$ |  | (1835) Eugenio BELTRAMI |
|  |  |  | (1597) Henry GELLIBRAND (1717) Jean Le Rond D'ALEMBERT (1790) August Ferdinand MOBIUS |
|  | $\begin{array}{cc} 18 & \mathrm{~W} \\ 19 & \mathrm{~T} \end{array}$ |  | (1872) Giovanni Enrico Eugenio VACCA (1927) Jon Leslie BRITTON |
|  |  |  | (1894) Heinz HOPF <br> (1900) Michail Alekseevich LAVRENTEV <br> (1901) Nina Karlovna BARI |
|  | 20 | F | (1889) Edwin Powell HUBBLE (1924) Benoit MANDELBROT |
|  | 21 | $\begin{aligned} & \mathrm{S} \\ & \mathrm{~S} \end{aligned}$ | (1867) Dimitri SINTSOV |
|  | 22 |  | (1803) Giusto BELLAVITIS <br> (1840) Emile Michel Hyacinte LEMOINE |
| 48 | $\begin{aligned} & 23 \\ & 24 \\ & 25 \\ & 26 \\ & 27 \\ & 28 \\ & 29 \end{aligned}$ | $\begin{aligned} & \mathrm{M} \\ & \mathrm{~T} \\ & \mathrm{~W} \\ & \mathrm{~T} \\ & \mathrm{~F} \\ & \mathrm{~S} \\ & \mathrm{~S} \end{aligned}$ | (1616) John WALLIS <br> (1820) Issac TODHUNTER |
|  |  |  | (1549) Duncan MacLaren Young SOMERVILLE (1909) Gerhard GENTZEN |
|  |  |  | (1873) Claude Louis MATHIEU <br> (1841) Fredrich Wilhelm Karl Ernst SCHRODER |
|  |  |  | (1894) Norbert WIENER <br> (1946) Enrico BOMBIERI |
|  |  |  | (1867) Arthur Lee DIXON |
|  |  |  | (1898) John WISHART |
|  |  |  | (1803) Christian Andreas DOPPLER <br> (1849) Horace LAMB <br> (1879) Nikolay Mitrofanovich KRYLOV |
| 49 | 30 | M | (1549) Sir Henry SAVILE |

## USAMO 2000, Pr. 5

Let $A_{1} A_{2} A_{3}$ be a triangle and let $\omega_{1}$ be a circle in its plane passing through $A_{1}$ and $A_{2}$. Suppose there exist circles $\omega_{2}, \omega_{3}, \ldots, \omega_{7}$ such that for $k=2,3, \ldots, 7, \omega_{k}$ is externally tangent to $\omega_{k-1}$ and passes through $A_{k}$ and $A_{k+1}$, where $A_{n+3}=A_{n} \forall n \geq 1$.

Prove that $\omega_{7}=\omega_{1}$.

## Mathematic Terms

QUANTIFY: I can't find anything wrong with your proof except that it won't work if $x$ is a moon of Jupiter (Popular in applied math courses).
PROOF OMITTED: Trust me, It's true.

## Set Theory

Theorem: Consider the set of all sets that have never been considered.
Hey! They're all gone!! Oh, well, never mind...
"Of the many forms of false culture, a premature converse with abstractions is perhaps the most likely to prove fatal to the growth of a masculine vigour of intellect."

George BOOLE
"A scientist can hardly meet with anything more undesirable than to have the foundations give way just as the work is finished. I was put in this position by a letter from Mr. Bertrand Russell when the work was nearly through the press."

Fredrich Ludwig Gottlob FREGE
"Logic is the hygiene the mathematician practices to keep his ideas healthy and strong."

Hermann Klaus Hugo WEYL
"The British Mathematical Colloquium consists of three days of mathematics with no dogs and no wives."

John Henry Constantine WHITEHEAD
"The modern physicist is a quantum theorist on Monday, Wednesday, and Friday and a student of gravitational relativity theory on Tuesday, Thursday, and Saturday. On Sunday he is neither, but is praying to his God that someone, preferably himself, will find the reconciliation between the two views."

Benoit MANDELBROT
"Algebra is generous: she often gives more than is asked for."

Jean D'ALEMBERT


## Rudi Mathematici

## December

|  |  | T | (1792) Nikolay Yvanovich LOBACHEVSKY |
| :---: | :---: | :---: | :---: |
|  | 2 | W | (1831) Paul David Gustav DU BOIS-RAYMOND <br> (1901) George Frederick James TEMPLE |
|  |  | T | (1903) Sidney GOLDSTEIN <br> (1924) John BACKUS |
|  |  | F | (1795) Thomas CARLYLE |
|  | 5 | S | (1868) Arnold Johannes Wilhelm SOMMERFELD <br> (1901) Werner Karl HEISENBERG |
|  | 6 | S | (1682) Giulio Carlo FAGNANO dei Toschi |
| 50 | 7 | M | (1647) Giovanni CEVA <br> (1823) Leopold KRONECKER <br> (1830) Antonio Luigi Gaudenzio Giuseppe CREMONA |
|  | 8 | T | (1508) Regnier GEMMA FRISIUS <br> (1865) Jaques Salomon HADAMARD <br> (1919) Julia Bowman ROBINSON |
|  | 9 | W | (1883) Nikolai Nikolaievich LUZIN <br> (1906) Grace Brewster MURRAY HOPPER <br> (1917) Sergei Vasilovich FOMIN |
|  | 10 | T | (1804) Karl Gustav Jacob JACOBI <br> (1815) Augusta Ada KING Countess of LOVELACE |
|  | 11 | F | (1882) Max BORN |
|  |  | S | (1832) Peter Ludwig Mejdell SYLOW |
|  | 13 | S | (1724) Franz Ulrich Theodosius AEPINUS <br> (1887) George POLYA |
| 51 | $\begin{aligned} & 14 \\ & 15 \\ & 16 \\ & 17 \end{aligned}$ | $\begin{aligned} & \mathrm{M} \\ & \mathrm{~T} \\ & \mathrm{~W} \\ & \mathrm{~T} \end{aligned}$ | (1546) Tycho BRAHE |
|  |  |  | (1802) Janos BOLYAI |
|  |  |  | (1804) Wiktor Yakovievich BUNYAKOWSKY |
|  |  |  | (1706) Gabrielle Emile Le Tonnelier de Breteuil du CHATELET <br> (1835) Felice CASORATI <br> (1842) Marius Sophus LIE <br> (1900) Dame Mary Lucy CARTWRIGHT |
|  | $\begin{aligned} & 18 \\ & 19 \\ & 20 \end{aligned}$ | FSS | (1917) Roger LYNDON |
|  |  |  | (1783) Charles Julien BRIANCHON (1854) Marcel Louis BRILLOUIN |
|  |  |  | (1494) Oronce FINE <br> (1648) Tommaso CEVA <br> (1875) Francesco Paolo CANTELLI |
| 52 | 2122 | $\begin{aligned} & \mathrm{M} \\ & \mathrm{~T} \end{aligned}$ | (1878) Jan LUKASIEVIKZ <br> (1932) John Robert RINGROSE |
|  |  |  | (1824) Francesco BRIOSCHI <br> (1859) Otto Ludwig HOLDER <br> (1877) Tommaso BOGGIO <br> (1887) Srinivasa Aiyangar RAMANUJAN |
|  | $\begin{array}{ll} 23 & \text { W } \\ 24 & \text { T } \end{array}$ |  | (1872) Georgii Yurii PFEIFFER |
|  |  |  | (1822) Charles HERMITE <br> (1868) Emmanuel LASKER |
|  | 25 | F | (1642) Isaac NEWTON <br> (1900) Antoni ZYGMUND |
|  | 26 | S | (1780) Mary Fairfax Greig SOMERVILLE <br> (1791) Charles BABBAGE |
|  | 27 | S | (1571) Johannes KEPLER <br> (1654) Jacob (Jacques) BERNOULLI |
| 1 | 28 | M | (1808) Athanase Louis Victoire DUPRĖ (1882) Arthur Stanley EDDINGTON (1903) John von NEUMANN |
|  | 29 T <br> 30 W <br> 31 T |  | (1856) Thomas Jan STIELTJES |
|  |  |  | (1897) Stanislaw SAKS |
|  |  |  | (1872) Volodymyr LEVIYTSKY <br> (1896) Carl Ludwig SIEGEL <br> (1952) Vaughan Frederick Randall JONES |

USAMO 200, Pr. 6
Let $a_{1}, b_{1}, a_{2}, b_{2}, \ldots, a_{n}, b_{n}$ be nonnegative real numbers. Prove that

$$
\sum_{i, j=1}^{n} \min \left\{a_{i} a_{j}, b_{i} b_{j}\right\} \leq \sum_{i, j=1}^{n} \min \left\{a_{i} b_{j}, a_{j} b_{i}\right\}
$$

## Mathematic Terms

Obvious: The instructor is sure it is in his notes somewhere.

Certainly: The instructor saw one of his instructors do it, but has completely forgotten how it was done.

## Statistics

Q: Did you hear the one about the statistician?
A: Probably....
"The shortest path between two truths in the real domain passes through the complex domain."

Jaques Salomon HADAMARD
"Now it is quite clear to me that there are no solid spheres in the heavens, and those that have been devised by authors to save the appearances, exist only in their imagination, for the purpose of permitting the mind to conceive the motion which the heavenly bodies trace in their courses."

Tycho BRAHE
"Mathematical discoveries, like springtime violets in the woods, have their season which no human can hasten or retard."

Janos BOLYAI
"I believe there are 15747724136275002577 605653961181555468044717914527116 709366231425076185631031296296 protons in the universe and the same number of electrons."

Arthur EDDINGTON
"The Analytical Engine weaves algebraic patterns, just as the Jacquard loom weaves flowers and leaves."
Augusta Ada KING Countess of LOVELACE "An expert is someone who knows some of the worst mistakes that can be made in his subject, and how to avoid them."

Werner Karl HEISENBERG

